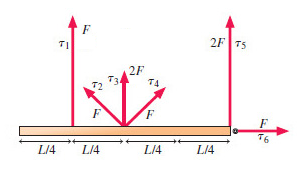
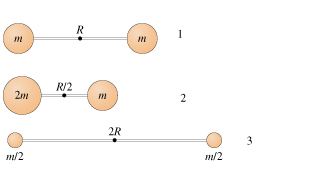
**Question 1.** Suppose a door rotates about a hinge, on the *right*. Rank the torques from smallest to largest. You can assume the angles that τ­2 and τ4 make with the rod are 45°.



(τ6,τ5), (τ4, τ2), τ1, τ3.

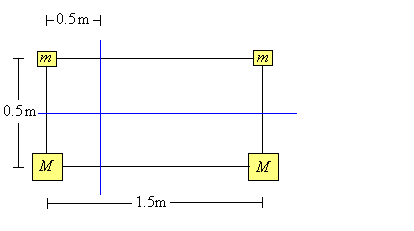
**Question 2**. Rank the moments of inertia from smallest to largest.



I1 = 2m(R/2)2 = mR2/2, I2 = m(R/4)2 + (2m)(R/4)2 = 3mR2/16, I3 = (m/2)R2 + (m/2)R2 = mR2. So order is I2, I1, I3.

**Problem 8.31**

Calculate the moment of inertia of the array of objects shown about the (a) the vertical axis, and (b) the horizontal axis. Assume m = 1.8 kg, and M = 3.1 kg. About which axis would it be harder to accelerate this array?

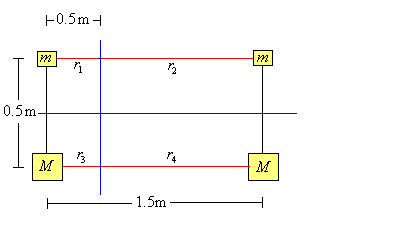


**Solution**

To calculate the moment of inertia about any axis, we use the formula:



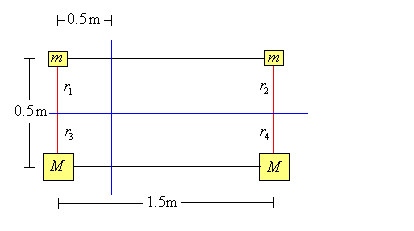
meaning we add up the product mr2 for each mass, where r is the distance between the mass and the axis of rotation. So let’s consider the vertical axis first. The distances from this axis to the masses are shown in red.



So…



Now consider the horizontal axis. Again, the r’s are shown in red.

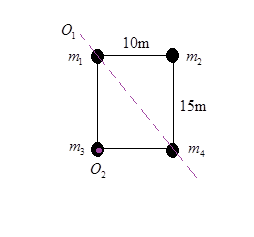


and so we have,

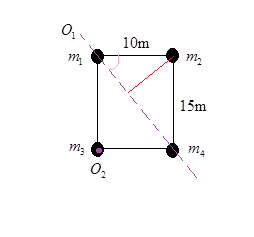


and so since Ihorizontal < Ivertical, we see it would be easier to rotate the masses about the horizontal axis.

5a. Consider the following object. Let m1 = 1kg, m2 = 2kg, m3 = 3kg, and m4 = 4kg. What is the moment of inertia of this object about axis O1, and about axis O2 (which passes through mass m3 perpendicular to the page)?



Note that the distance between m2 and m3 from the axis O1 is given by the red line. The pink angle is θ = tan-1(15/10) = 56.3°. And so the red line distance is r = 10sin(56.3) = 8.3m.



and so the moment of inertia is:

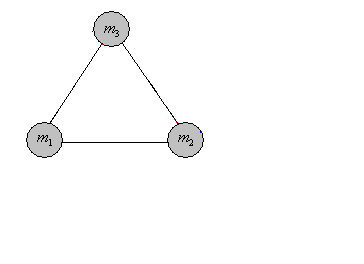


About O2 we have:

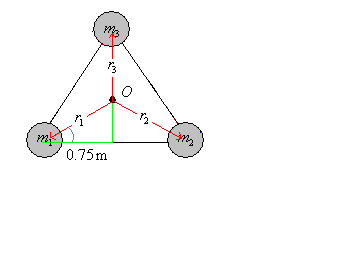


**Example: Calculating I some more**

Consider the following equilateral triangle (side length 1.5m), with m1 = 1kg, m2 = 2kg, and m3 = 3kg.



What is the moment of inertia about an axis passing through the center of the triangle? The center is shown below. The radii r1, r­2, r3 are all equal to each other. To calculate them, we can draw in the following triangle (in green/red).



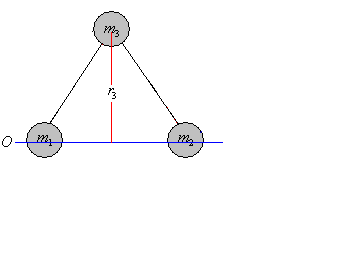
The base of the triangle is ½ the distance from m1 to m2 (0.75m), and the angle illustrated is ½ of 60˚. So using some trig, r1, the hypotenuse is given by:



So the moment of inertia is:



What is the moment of inertia about an axis passing through m1 and m­2?



In this case, r3 is given by,



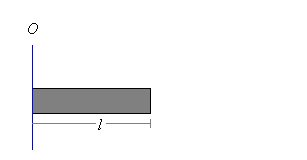
and the moment of inertia is:



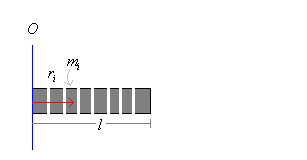
Comparing the two moments of inertia, we see that it is easier to rotate the mass triangle about the first axis, than about the second.

**Example: Calculating I for a board about end point**

Consider a continuous object, a board of length ℓ, with mass, M. What is the moment of inertia of the board about an axis, O, passing through an end point?



Again, we’d have to break the board up into little pieces, calculate mr2 for each, and then add them up. A typical mi­ and r­­i are shown below,



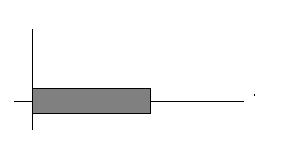
I is calculated via:



As we break the object up into finer and finer pieces, the sum becomes an integral. And we get,



8. Consider a board with length 2m and linear mass density ρ(x) = 2 + x4. What is the moment of inertia, I, of the board about the origin?

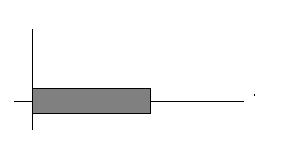


This is;



**Question 3**

Consider a board with length 5m and linear mass density λ(x) = 1+ x2. What is the moment of inertia, I, of the board about the origin?



We just have to integrate…



**Example: Calculating I for a board about its center**

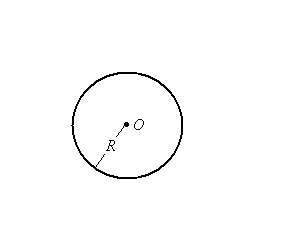
What is the moment of inertia of such uniform board about its center? Then we put O at the center, and measure distances from there.



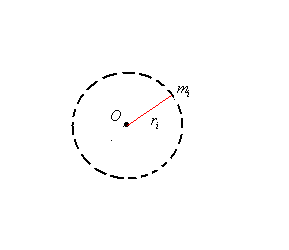
In similar fashion, one can calculate the moment of inertia of other objects.

**Example: Calculating I for hoop**

Consider a hoop of radius R, and mass M. What is its moment of inertia about its center?



To determine this, we must break the object into little pieces. Calculate mr2 for each piece, and then add. A particular m­­i and its distance from O, ri is shown below.



The moment of inertia about O would then be calculated via:

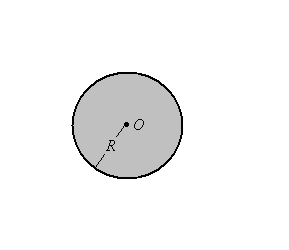


So the moment of inertia of the hoop about its center is:



**Example: Calculating I for disk about center**

Using the same procedures we can determine, in principle, the moments of inertia of other objects. Consider a solid disk. We can calculate the moment of inertia of the disk about its center:



It is (calling σ the mass density)



Now the total mass is M = σA = σπR2 and so we have σ = M/πR2. Filling this in…



So:



**Example: Moment of inertia of sphere about its center**

Doing similar calculations we can show that:



For instance, let O be the axis running through the center of the sphere. Then,



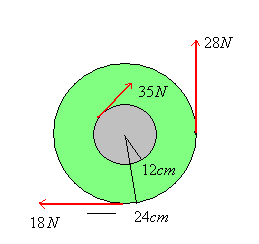
Finally, the mass is M = ρ(4/3)πR3. So filling this in:



so there we are.

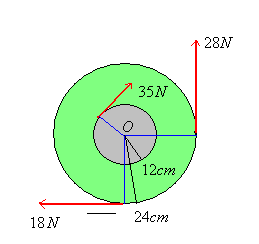
**Problem 8.24**

Calculate the net torque about the axle of the wheel show in Fig. 8-39. Assume that a friction torque of 0.40 N·m opposes the motion.



**Solution**

*Remember* the torque produced by a force is . In reference to the , a counter-clockwise rotation (CCW) is considered positive, and so is a torque that would induce such a rotation. Likewise, a clockwise rotation is negative, and so is a torque which would induce such a rotation. r is the moment arm which points from the axis of rotation O to the point of application of the force, F. And θ is the angle between r and F. They tell us to calculate the torque about the axis of the wheel – i.e. the center of the wheel. So we draw all moment arms from the center of the wheel to the where the particular force is being applied (in blue).



Then we add up the torques.



Note the 135˚ angle they show in the text isn’t relevant. Finally, they say that τf is a 0.40 N·m torque which opposes motion that the other forces are trying to set up. These forces are setting up a CW rotation (since their net torque is -1.8 N·m). So the sign of τ, which we wouldn’t know until now, is +. So we have,

